

6.3 Assess Your Understanding

Concepts and Vocabulary

- $\tan \frac{\pi}{4} + \sin 30^\circ =$ _____
- Using a calculator, $\sin 2 =$ _____, rounded to two decimal places.
- True or False:* Exact values can be found for the trigonometric functions of 60° .
- True or False:* Exact values can be found for the sine of any angle.

Exercises

- Write down the exact value of each of the six trigonometric functions of 45° .
- Write down the exact value of each of the six trigonometric functions of 30° and of 60° .

In Problems 7–16, find the exact value of each expression if $\theta = 60^\circ$. Do not use a calculator.

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|-----------------------|---------------------|----------------------------|-----------------------------|-----------------------------|
| 7. $\sin \theta$ | 8. $\cos \theta$ | 9. $\sin \frac{\theta}{2}$ | 10. $\cos \frac{\theta}{2}$ | 11. $(\sin \theta)^2$ |
| 12. $(\cos \theta)^2$ | 13. $2 \sin \theta$ | 14. $2 \cos \theta$ | 15. $\frac{\sin \theta}{2}$ | 16. $\frac{\cos \theta}{2}$ |

In Problems 17–28, find the exact value of each expression. Do not use a calculator.

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|---|---|---|
| 17. $4 \cos 45^\circ - 2 \sin 45^\circ$ | 18. $2 \sin 45^\circ + 4 \cos 30^\circ$ | 19. $6 \tan 45^\circ - 8 \cos 60^\circ$ |
| 20. $\sin 30^\circ \cdot \tan 60^\circ$ | 21. $\sec \frac{\pi}{4} + 2 \csc \frac{\pi}{3}$ | 22. $\tan \frac{\pi}{4} + \cot \frac{\pi}{4}$ |
| 23. $\sec^2 \frac{\pi}{6} - 4$ | 24. $4 + \tan^2 \frac{\pi}{3}$ | 25. $\sin^2 30^\circ + \cos^2 60^\circ$ |
| 26. $\sec^2 60^\circ - \tan^2 45^\circ$ | 27. $1 - \cos^2 30^\circ - \cos^2 60^\circ$ | 28. $1 + \tan^2 30^\circ - \csc^2 45^\circ$ |

In Problems 29–46, use a calculator to find the approximate value of each expression. Round the answer to two decimal places.

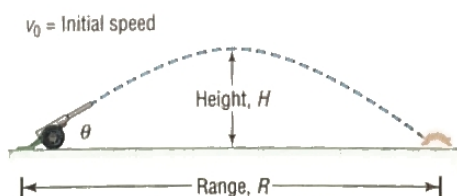
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|---------------------------|--------------------------|----------------------------|---------------------------|---------------------------|----------------------------|
| 29. $\sin 28^\circ$ | 30. $\cos 14^\circ$ | 31. $\tan 21^\circ$ | 32. $\cot 70^\circ$ | 33. $\sec 41^\circ$ | 34. $\csc 55^\circ$ |
| 35. $\sin \frac{\pi}{10}$ | 36. $\cos \frac{\pi}{8}$ | 37. $\tan \frac{5\pi}{12}$ | 38. $\cot \frac{\pi}{18}$ | 39. $\sec \frac{\pi}{12}$ | 40. $\csc \frac{5\pi}{13}$ |
| 41. $\sin 1$ | 42. $\tan 1$ | 43. $\sin 1^\circ$ | 44. $\tan 1^\circ$ | 45. $\tan 0.3$ | 46. $\tan 0.1$ |

Projectile Motion The path of a projectile fired at an inclination θ to the horizontal with initial speed v_0 is a parabola (see the figure). The range R of the projectile, that is, the horizontal distance that the projectile travels, is found by using the formula

$$R = \frac{2v_0^2 \sin \theta \cos \theta}{g}$$

where $g \approx 32.2$ feet per second per second ≈ 9.8 meters per second per second is the acceleration due to gravity. The maximum height H of the projectile is

$$H = \frac{v_0^2 \sin^2 \theta}{2g}$$



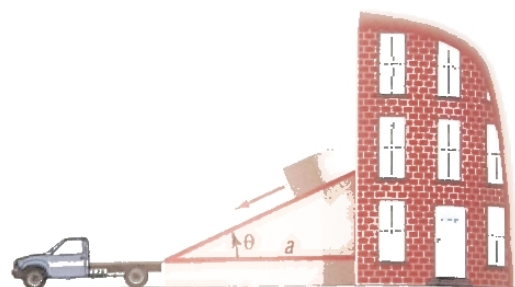
In Problems 47–50, find the range R and maximum height H . Round answers to two decimal places.

47. The projectile is fired at an angle of 45° to the horizontal with an initial speed of 100 feet per second.
48. The projectile is fired at an angle of 30° to the horizontal with an initial speed of 150 meters per second.
49. The projectile is fired at an angle of 25° to the horizontal with an initial speed of 500 meters per second.
50. The projectile is fired at an angle of 50° to the horizontal with an initial speed of 200 feet per second.
51. **Inclined Plane** If friction is ignored, the time t (in seconds) required for a block to slide down an inclined plane (see the figure) is given by the formula

$$t = \sqrt{\frac{2a}{g \sin \theta \cos \theta}}$$

where a is the length (in feet) of the base and $g \approx 32$ feet per second per second is the acceleration of gravity. How long does it take a block to slide down an inclined plane with base $a = 10$ feet when

(a) $\theta = 30^\circ$? (b) $\theta = 45^\circ$? (c) $\theta = 60^\circ$?



52. **Piston Engines** In a certain piston engine, the distance x (in meters) from the center of the drive shaft to the head of the piston is given by

$$x = \cos \theta + \sqrt{16 + 0.5(2 \cos^2 \theta - 1)}$$

where θ is the angle between the crank and the path to the piston head (see the figure). Find x when $\theta = 30^\circ$ and when $\theta = 45^\circ$.



53. **Calculating the Time of a Trip** Two oceanfront houses are located 8 miles apart on a straight stretch of beach, each a distance of 1 mile from a paved road that parallels the ocean. Sally can jog 8 miles per hour along the paved road, but only 3 miles per hour in the sand on the beach. Because of a river between the two houses, it is necessary to jog on the sand to the road, continue on the road, and then jog on the sand to get from one house to the other. See the illustration.

